Adaptive Command Shaping Using Adaptive Filter Approach in Time Domain

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Abstract

Since its introductionΓthe command shaping method has been applied to the control of many different types of flexible manipulators. A properly designed command shaper cancels the resonance poles of the system regardless of the given reference input to the system. HoweverΓ designing an effective command shaper requires *a priori* knowledge about the system parameters. RecentlyΓ some efforts have been made to make the command shaper less sensitive to the uncertainty of the system parameters and to make the command shaper adapt to the unknown system parameters. This research is an effort to develop a effective adaptive command-shaping algorithm in the time domain. In this paperΓthe authors propose an adaptive command-shaping algorithm using adaptive filtering technique in the time domain and verify the effectiveness of the proposed algorithm with proper experiments.

1 Introduction

Having flexibility in a manipulator will degrade trajectory tracking control and manipulator tip positioning. In practiceΓhoweverΓconstraints imposed by various operating requirementsΓwill render the presence of such flexibility unavoidable. Numerous researchers have proposed control schemes to attenuate tip vibration due to flexure during mechanical manipulator motion. A feedforward approachΓthe command shaping techniqueΓ seeks to reduce tip vibrations by reshaping the desired trajectory not to excite the resonances of the flexible manipulator. It accomplishes this task without regard to the particular form of the desired trajectory. Since its introduction [1]Γthe command shaping method has been applied successfully in many different applications. UnfortunatelyΓ howeverΓ a proper design of command shaper requires the knowledge of the flexible system parameters such as natural frequency and damping ratio of the undesired elastic mode.

When the command shaping method is used for flexible manipulator controlΓthere are two ways of handling the uncertainty of the system parameters. The first one is the robust design of the command shaper based on the limited known system information to make it less sensitive to the uncertainty of the system parameters. Unfortunately this would end up increasing the command shaper lengthΓ which means more delay in the response. MoreoverΓ this technique still needs fair a amount of *a priori* knowledge about the system parameters. The second one is to make the command shaper adapt to the uncertainty of the flexible system. Until nowΓall proposed adaptive command shaping methods are based on the system estimation approach either in the frequency-domain [2]Γ or in the time domain ([3]Γ [4]). In these algorithmsΓnew filter coefficients are calculated using the system estimation results. For the frequency-domain approachΓeven though there are several efficient algorithms developedΓit is still a computational burden to do a FFT analysis during the control calculation. The system estimation in the time domain often gives a poor estimation of the system. Consequently so does the adaptation of the command shaper relying on the accuracy of system estimation results.

In this paperΓthe authors propose an adaptive filtering method to adapt a command shaper to directly minimize the residual vibration instead of the system estimation error. When we adapt a finite impulse response (FIR) type command shaper without knowing the system parametersΓ two kinds of things must be adapted: the locations of impulses and the magnitude of the impulses. It is known that the adaptation of impulse locations of an FIR filter is a very complex nonlinear problem. FortunatelyΓhoweverΓa three impulse command shaper called optimal arbitrary time-delay filter (OATF) renders the freedom in choosing the impulse location regardless of the system parameters [5]. In other wordsΓif the impulse magnitudes of the OATF are properly decidedΓwe can choose any location of the impulses to cancel out the given elastic mode. ThereforeΓ using the structure of the OATFΓ we only need to adapt the magnitude of the impulses of a command shaper with fixed locations. At the end of this paperΓ experiment results using a gantry type robot are listed to show the effectiveness of the proposed algorithm.

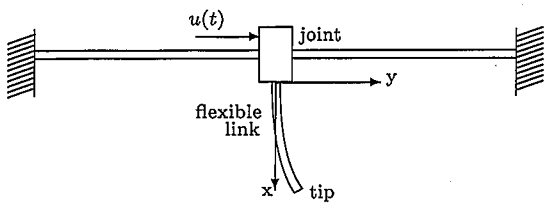


Figure 1: Schematic Diagram of the Gantry Robot with a Flexible Link.

2 Dynamics of Flexible Manipulator

The gantry type robot with a prismatic joint and a flexible manipulator conceptually shown in Figure 1 is one of the most popular systems used in the industry. Even though the proposed method can be used with the other types of flexible manipulatorsΓ this gantry type robot is adopted as the initial test bed for this study. For the dynamic analysis of this systemΓthe assumed mode method and Lagrange method are used to obtain the equations of motions. The assumed modes method represents the system response $y(t,x)$ with finite number of assumed modes $\phi_n(y)$ and generalized coordinates $q_i(t)$ as shown in (1).

$$y(t,x) = \sum_{i=1}^{N} q_i(t)\phi_i(x)\Gamma \qquad (1)$$

ThenΓby using Lagrange's equationsΓwe find the system's equations of motion as shown in (2).

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{D}\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) = \boldsymbol{B}u(t)\Gamma \qquad (2)$$

whereΓ

$$\boldsymbol{M} \equiv \begin{bmatrix} M_r & \boldsymbol{M}_{re} \\ \boldsymbol{M}_{re}^T & \boldsymbol{M}_e \end{bmatrix}\Gamma\ \boldsymbol{K} \equiv \begin{bmatrix} 0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_e \end{bmatrix}\Gamma$$

$$\boldsymbol{D} = \alpha\boldsymbol{M} + \beta\boldsymbol{K}\Gamma\ \ \boldsymbol{B} = \begin{bmatrix} 1 \\ \boldsymbol{0} \end{bmatrix}\Gamma$$

α and β are constants (proportional damping is assumed)Γand $\boldsymbol{q}(t) = [q_r(t)\ \boldsymbol{q}_e(t)]^T$Γwhere $q_r(t)$ is the rigid generalized coordinate (representing the joint motion) and $\boldsymbol{q}_e(t)$ are the flexible generalized coordinates (representing the flexible link motion). The subscripts r and e in the above equations denote rigid body motion and elastic motionΓrespectively; $\boldsymbol{M}_{re}$ represents coupling between the rigid mode and the elastic modes. Only one rigid mode appears in the above equation corresponding to the horizontal translation. GenerallyΓthe mass matrix $\boldsymbol{M}$ and the stiffness matrix $\boldsymbol{K}$ may vary with the system configuration $\boldsymbol{q}(t)$Γbut for our gantry-type manipulator with fixed beam lengthΓthese matrices are constant. The manipulator joint displacementΓ$y_j(t)$Γand the tip displacementΓ$y_t(t)$Γare expressed by equation (3).

$$\boldsymbol{y}(t) \equiv \begin{bmatrix} y_j(t) \\ y_t(t) \end{bmatrix} = \begin{bmatrix} \Phi^T(0) \\ \Phi^T(L) \end{bmatrix} \boldsymbol{q}(t)\Gamma \qquad (3)$$

where $\Phi(z)^T = [\phi_1(z)\ \phi_2(z)\ \cdots\ \phi_N(z)]$.

3 Command Shaping

The *command shaper* reshapes the desired input to a flexible system such that the resonances of the elastic system modes are not excited. It takes the form of an FIR filterΓwith filter parameters determined by the resonant frequencies and the damping ratios of the undesired elastic modes of the flexible system. In this research we have used a particular *three-term* command shaper called the *optimal arbitrary time-delay filter* (OATF). For a single elastic mode cancellationΓan OATF is given by the following equation:

$$c(t) \triangleq \frac{1}{M}(\delta(t) - 2\cos(\omega_d T_d)e^{-\zeta\omega_n T_d}\delta(t - T_d) + e^{-2\zeta\omega_n T_d}\delta(t - 2T_d))\Gamma \quad (4)$$

where T_d is the time delayΓ$\delta(t)$ is the unit impulse function centered at $t = 0$Γω_n is the natural frequency of the undesired elastic modeΓζ is the corresponding damping ratioΓω_d is the corresponding damped natural frequencyΓand $M \triangleq 1 - 2\cos(\omega_d T_d)e^{-\zeta\omega_n T_d} + e^{-2\zeta\omega_n T_d}$. In order to have the same total steady-state response both before and after the command shaping of the inputΓthe command shaper is normalized to have unit DC gain.

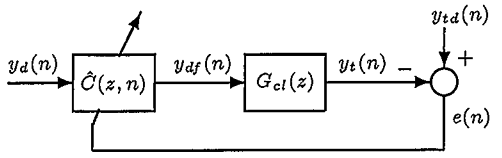
It has been shown that if the command shaper coefficients are properly chosenΓthe OATF is capable of canceling the given resonance poles with its zeros using any T_d; note that this is not true for earlier command shaping methods. The equation (5) gives the zeros of the OATF in the s-domain. Using any value of T_dΓthis filter has an infinite number of zerosΓincluding the zeros at the locations of the resonance poles of the flexible system:

$$s = -\zeta\omega_n \pm j(\omega_d + \frac{2n\pi}{T_d})\Gamma \qquad (5)$$

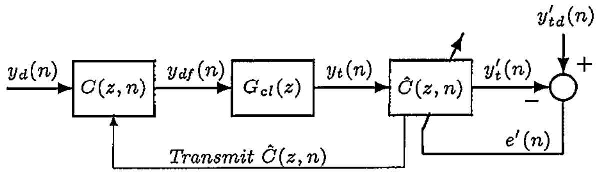
where $n = 0, \pm1, \pm2, \ldots$. When we realize command shaping in the discrete-time domainΓthe time delay T_d is not permitted to be an arbitrary number. InsteadΓit must be chosen as an integer multiple of the plant sampling timeΓT_s. The freedom of the OATF in choosing the time-delay makes it easy to implement in a digital control system. The z-domain representation of the OATF is given by

$$C(z) \triangleq \frac{1}{M}(1 - 2\cos(\omega_d T_d)e^{-\zeta\omega_n T_d}z^{-\Delta} + e^{-2\zeta\omega_n T_d}z^{-2\Delta})\Gamma \quad (6)$$

where $\Delta = \mathit{integer} = T_d/T_s$.



(a) Pre Adaptive Filtering Configuration



(b) Post Adaptive Filtering Configuration

Figure 2: Adaptive Command Shaping Configurations.

4 Adaptive Command Shaping

As a direct approach to adapt the command shaperΓ an adaptive filtering approach is proposed here. The adaptive command shaper $\hat{C}(z,n)$ has the form of the OATF as shown in (7) .

$$\hat{C}(z,n) = \hat{a}_0(n) + \hat{a}_1(n)z^{-\Delta} + \hat{a}_2(n)z^{-2\Delta}\Gamma \quad (7)$$

where $\sum_{i=0}^{2} \hat{a}_i = 1$ (unity gain constraint). From the command shaper study in the previous section we know thatΓeven without knowing the system parametersΓwe can fix the time-delay T_d or Δ of equation (7) to cancel a given elastic mode. With fixed Δ and the unity gain constraintΓonly 2 coefficients are left to be adapted for a single mode vibration cancellation.

Figure 2 shows two conceivable configurations for an adaptive command shaper (ACS) using the adaptive filtering approach. In Figure 2Γthe adaptation error $e(n)$ and $e'(n)$ drive the adaptation of $\hat{C}(z,n)$ at time n where $y_d(n)$ is a desired reference inputΓ$y_{df}(n)$ is a filtered reference input signalΓ$G_{cl}(z)$ is a closed loop joint control system of a flexible manipulatorΓ$y_t(z)$ is actual tip response and $y_{td}(n)$ is a desired tip response.

NowΓlet us consider the configuration (a) in Figure 2 first. The adaptation error $e(n)$ can be described in a form of the linearized parameter model as shown in (8).

$$\begin{aligned} e(n) &\equiv y_{td}(n) - y_t(n) \\ &= y_{td}(n) - \hat{C}(z,n)G_{cl}(z)y_d(n) \\ &= y_{td}(n) - W^T(n)\Theta(n)\Gamma \end{aligned} \quad (8)$$

where $\Theta(n) = [\ \hat{a}_0(n)\ \ \hat{a}_0(n)\ \ \hat{a}_0(n)\ ]^T\Gamma\ W(n) = [\ x(n)\ \ x(n-\Delta)\ \ x(n-2\Delta)\ ]^T$Γand $x(n) = G_{cl}(z)y_d(n)$. The regressor vector $W(n)$ which is indispensable for the adaptation algorithm requires $x(n)$ which is the tip response due to the unfiltered reference input $y_d(n)$ as you see in the above equation. UnfortunatelyΓhoweverΓ there is no way to get $x(n)(= G_{cl}(z)y_d(n))$ in the configuration (a). Only the tip response due to the filtered referenceΓ$G_{cl}(z)y_{df}(n)$Γis available.

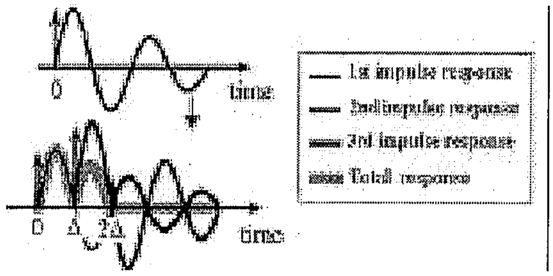


Figure 3: Elastic Response with a Three Term Command Shaper for an Impulse Input.

AlternativelyΓ let's think about the configuration shown in (b). If the command shaper and the flexible system are linearΓthen this swapped order of $G_{cl}(z)$ and $\hat{C}(z)$ will give the same optimal command shaper as the configuration (a) would do. In this configuration the adapted command shaper $\hat{C}(z,n)$ is copied to $C(z,n)$ which actually reshapes the command that goes into the flexible manipulator system $G_{cl}(z)$. In the post adaptive filtering configurationΓthe adaptation error $e'(n)$ is described as (9).

$$\begin{aligned} e'(n) &= y_{td}(n) - \hat{C}(z,n)y_t(n) \\ &= y_{td}(n) - W'^T(n)\Theta(n)\Gamma \end{aligned} \quad (9)$$

where $W'(n) = [\,y_t(n)\ \ y_t(n-\Delta)\ \ y_t(n-2\Delta)\ ]^T$ and it doesn't matter if $y_t(n)$ is due to $y_d(n)$ or $y_{df}(n)$. This allows us to have $C(z,n)$ in the front of $G_{cl}(z)$ as shown in (b) of Figure 2. ThusΓwe choose the configuration (b) as our ACS configuration.

Caution must be paid to the desired response $y_{td}(n)$. The desired tip response in configuration (b) is defined as (10).

$$y_{td}(n) = C_{opt}(z)C_{opt}(z)G_{cl}(z)y_d(n), \quad (10)$$

whereΓ$C_{opt}(z)$ is desired optimal command shaper. The problem is that $y_{td}(n)$ cannot be decided without knowing $C_d(z)$ and $G_{cl}(z)$. This problem can be illustrated using Figure 3. When the impulse input is consideredΓ $C_d(z)$ makes the residual vibration (represented by the hatching line in the figure) to be zero after time 2Δ which is the length of the command shaper. HoweverΓ between time 0 and 2ΔΓthere is a non-zero transient response that depends on the elastic system and the command shaper. Due to this characteristicsΓwe utilize only the residual period signal (after 2Δ) for the adaptation. Since the input $y_d(n)$ is available and Δ is a value that we chooseΓthe residual period can be found out beforehand.

Among many types of adaptive algorithmsΓthe recursive least squares (RLS) method is used in this research. It is one of the most popular algorithmsΓbecause of its ease of analysisΓeffective performanceΓrelatively fast convergence rate and convergeny independent of input characteristics. In this ACS approachΓ howeverΓthe *linearly constrained recursive least squares* (LCRLS) algorithm should be used instead of standard RLSΓbecause the adaptive command shaper is required to satisfy the unit DC gain constraint shown in (11) in a vector form.

$$D^T \Theta(n) = 1, \tag{11}$$

where $D = [\ 1 \quad 1 \quad 1\ ]^T$. The cost function that is minimized during the adaptation is listed in (12)Γwhich is a direct indication of the amount of the undesired vibration of the system.

$$\varepsilon(n) = \sum_{i=1}^{n} \lambda^{n-1} |e'(i)|^2 \Gamma \tag{12}$$

where λ = forgetting factorΓ$0 < \lambda \leq 1$. FinallyΓnecessary recursive equations to find an updated ACS coefficient vector $\Theta(n)$ at each instant n are listed below in (13) – (16).

$$K(n) = \frac{P(n-1)W'(n)}{\lambda + W'^T(n)P(n-1)W'(n)} \tag{13}$$

$$\Gamma(n) = \lambda^{-1}[\Gamma(n-1) - K(n)W'^T(n)\Gamma(n-1)] \tag{14}$$

$$P(n) = \lambda^{-1}[P(n-1) - K(n)W'^T(n)P(n-1)] \tag{15}$$

$$\Theta(n) = \Gamma(n)/[D^T\Gamma(n)]\Gamma \tag{16}$$

where $P^{-1}(n) = \sum_{i=1}^{n} \lambda^{n-i} W'(i)W'(i)$. The initial condition for $P(0)$ can be decided in the same way as in the standard RLS method.

5 Experiment

The flexible manipulator we have used in our experiments is the gantry-type robot shown in Figure 4. The post adaptive filtering configuration has been implemented on this test bed. The manipulator system has one prismatic joint that constrains the motion of the base to be along the horizontal track of linear motor. The displacement of the base is measured with an encoder. We use an accelerometer attached to the tip of flexible link to measure horizontal tip vibration. In the experimentsΓtip acceleration is used to adapt the command shaper instead of the tip displacement. Since the tip acceleration represents the tip vibration just as well as the displacement does (except for the magnitude difference)Γit shouldn't make a difference. With a 4 kg payload affixed to its tipΓthe manipulator has a single dominant elastic mode with a natural frequency of approximately 16 Hz. We have implemented our PD joint controller digitallyΓusing a 1 kHz sample rate. A repetitive trapezoidal velocity trajectory shown in Figure 5 is used as the reference input $y_d(n)$.

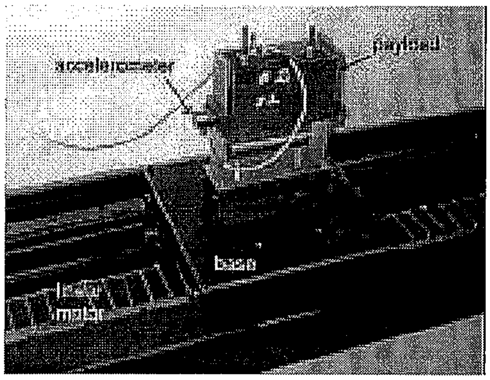


Figure 4: Picture of the Test Bed.

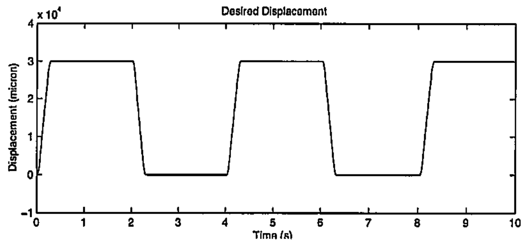


Figure 5: Reference Trajectory.

Assuming that we don't have knowledge about the flexible system parametersΓwe have tested several time-delay values for the ACS. All of them showed very similar resultsΓand only a single set of experimental results where the time-delay is set to be 25 msec are listed in this paper. In all those experimentsΓthe ACS algorithm is turned on at the beginning of the first residual period. The initial value of $P(0)$ has been chosen to be a diagonal matrixΓ$diag(p_0, p_0, p_0)$Γwhere $p_0 = 10^7$ and the initial value of $\Theta(0)$ that satisfies the recursive equations shown in the previous section is $[1/3 \ \ 1/3 \ \ 1/3]^T$. The transmission of the adapted command shaper coefficients of $\hat{C}(z,n)$ to $C(z,n)$ which actually filters the command has been initiated after one cycle of the trajectory which is 4 sec. After 4 secΓevery new ACS coefficients calculated are transmitted to $C(z,n)$ at every sample time during the residual period. Before the transmission beginsΓ$C(z,n)$ is kept at unityΓwhich means no command shaping.

Figure 6 shows the measured tip acceleration (top plot) and the adaptation of $\Theta(n)$ (bottom plot). We observe the significant reduction of the residual vibration down to the environmental noise level after the transmission of the ACS coefficients are initiated. Figure 7 shows the changes of the magnitude of the FFT of the ACS along the adaptation. Figure 8 is the contour map of Figure 7. Figure 9 is the magnitude plot of the FFT of the converged ACS at the end of adaptation. Figures 7 – 9 show the local minimum around 16 Hz.

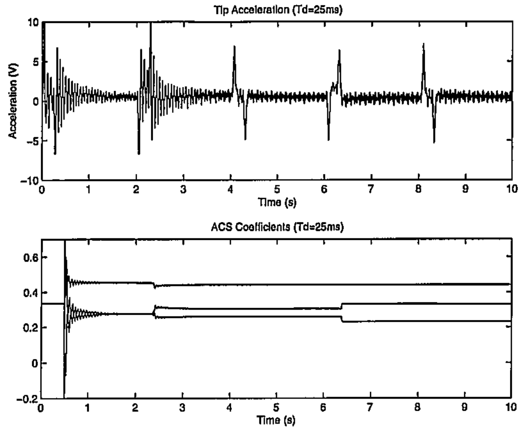


Figure 6: Change of the Tip Acceleration and the ACS Coefficients ($T_d = 25$ msec).

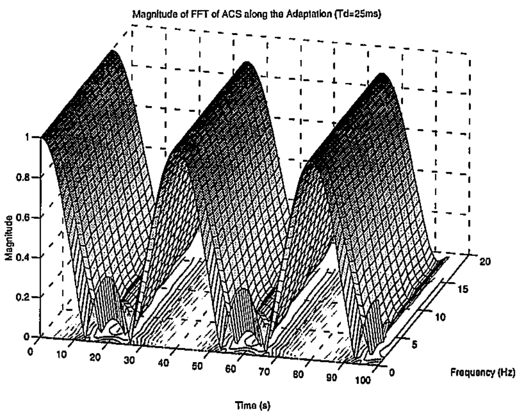


Figure 7: Three Dimensional Plot of Magnitude of FFT of ACS ($T_d = 25$ msec).

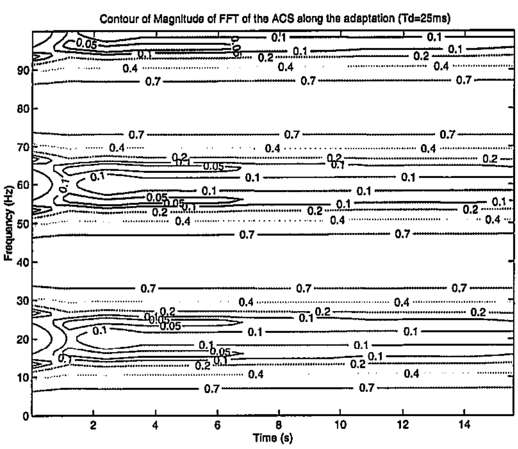


Figure 8: Contour of the Three Dimensional Magnitude Plot ($T_d = 25$ msec).

6 Conclusion

In this paperΓthe authors have proposed an adaptive command shaping method using the adaptive filtering approach in the time domain. We have shown experimentally that the proposed adaptive command shaping algorithm can effectively reduce the residual vibration of the unknown flexible manipulator system.

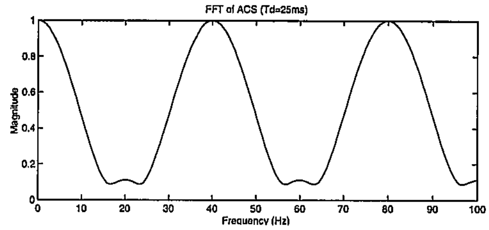


Figure 9: Magnitude of FFT of ACS ($T_d = 25$ msec).

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